Industrial Process Control
MDP 454

"If you have a smart project, you can say "I'm an engineer""

Lecture 4

Staff boarder

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Industrial Process Control
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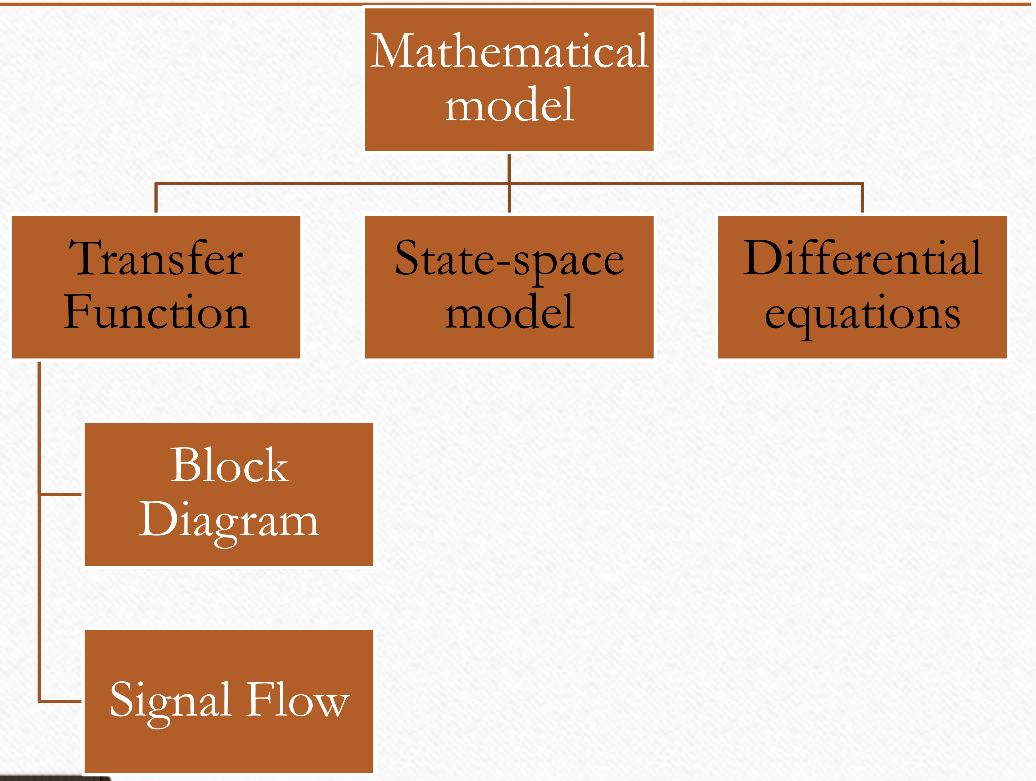
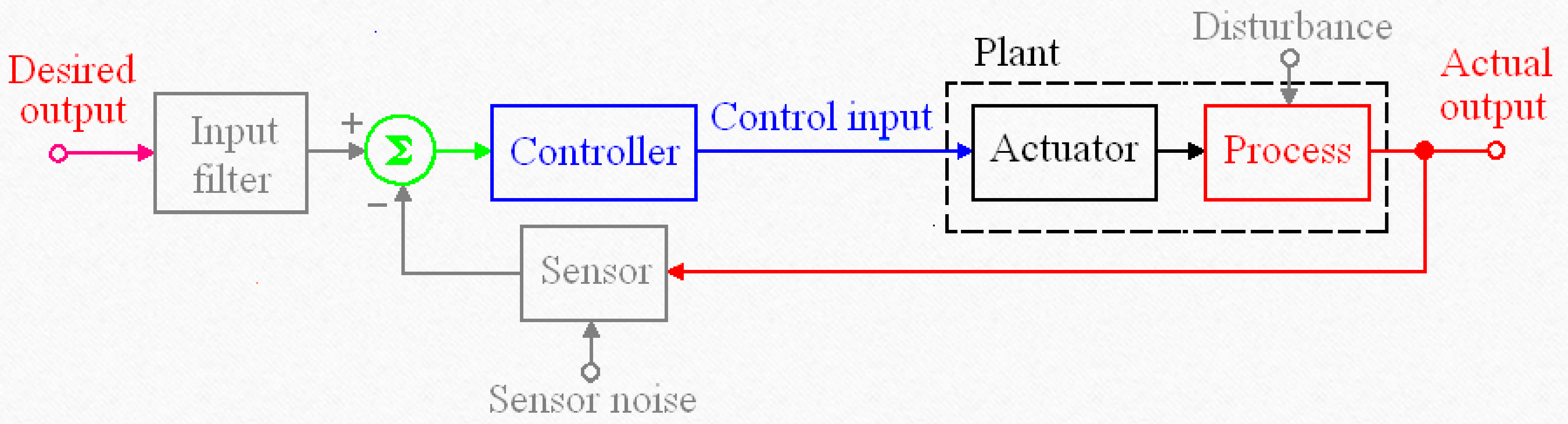
- Lecture aims:
  - Understand the Block reduction techniques
  - Identify the transfer function

Mathematical Modeling

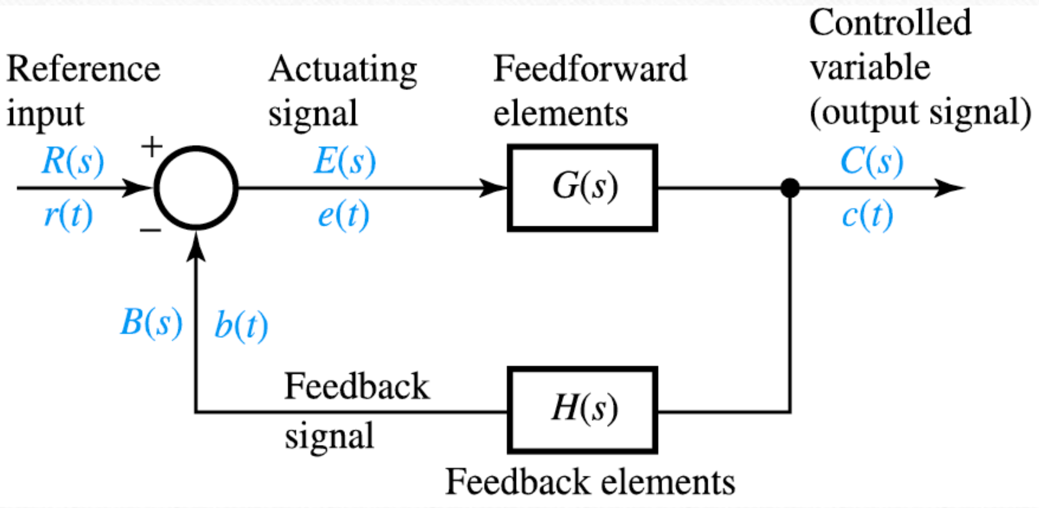
- Transfer Function



Component Block Diagram



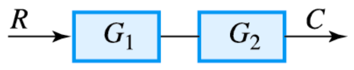
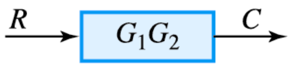
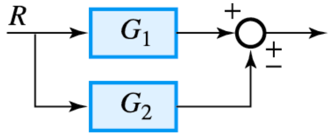
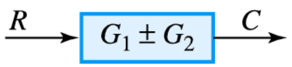
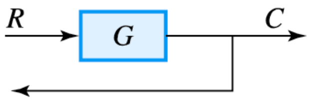
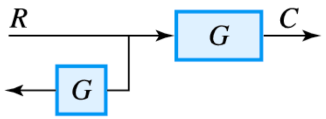
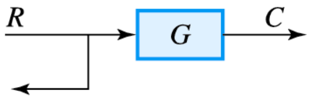
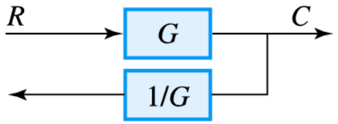
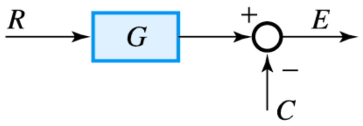
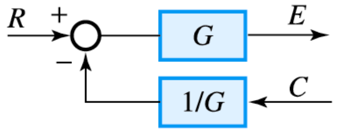
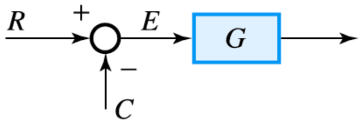
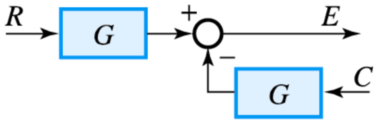
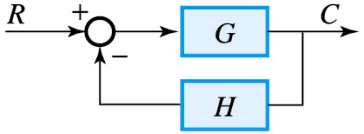
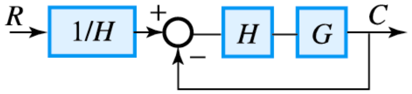
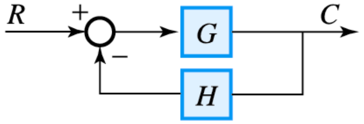
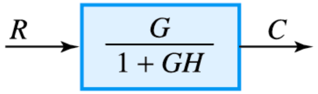
Component Block Diagram



$R(s)$	Reference input
$C(s)$	Output signal (controlled variable)
$B(s)$	Feedback signal = $H(s)C(s)$
$E(s)$	Actuating signal (error) = $[R(s) - B(s)]$
$G(s)$	Forward path transfer function or open-loop transfer function = $C(s)/E(s)$
$M(s)$	Closed-loop transfer function = $C(s)/R(s) = G(s)/[1 + G(s)H(s)]$
$H(s)$	Feedback path transfer function
$G(s)H(s)$	Loop gain

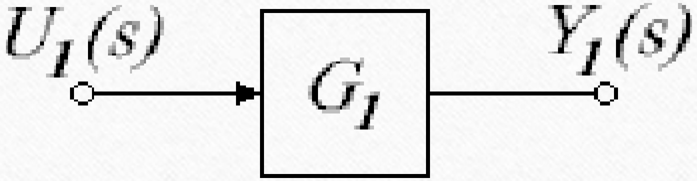
$$\frac{E(s)}{R(s)} = \text{Error-response transfer function } \frac{1}{1 + G(s)H(s)}$$

TABLE 3.4.1 Some of the Block Diagram Reduction Manipulations

Original Block Diagram	Manipulation	Modified Block Diagram
	Cascaded elements	
	Addition or subtraction (eliminating auxiliary forward path)	
	Shifting of pickoff point ahead of block	
	Shifting of pickoff point behind block	
	Shifting summing point ahead of block	
	Shifting summing point behind block	
	Removing H from feedback path	
	Eliminating feedback path	

Component Block Diagram

- It represents the ***mathematical relationships*** between the elements of the system.



$$U_1(s)G_1(s) = Y_1(s)$$

- The ***transfer function*** of each component is placed ***in box***, and the ***input-output relationships*** between components are indicated by ***lines and arrows***.

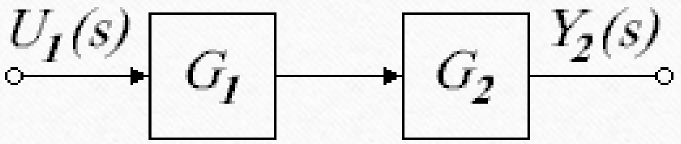
Component Block Diagram

- We can ***solve the equations by graphical simplification***, which is often easier and more informative than algebraic manipulation, even though the methods are in every way equivalent.

- The interconnections of blocks include summing points, where any number of signals may be added together.

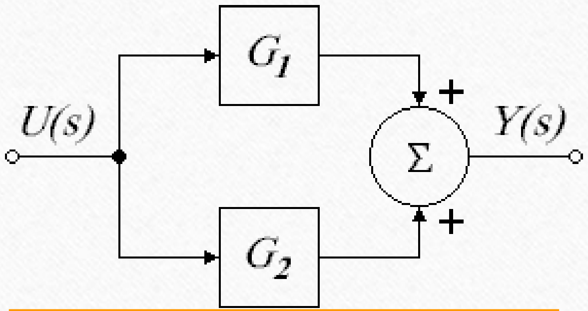
Block Diagram Reduction Technique

- Blocks in series:



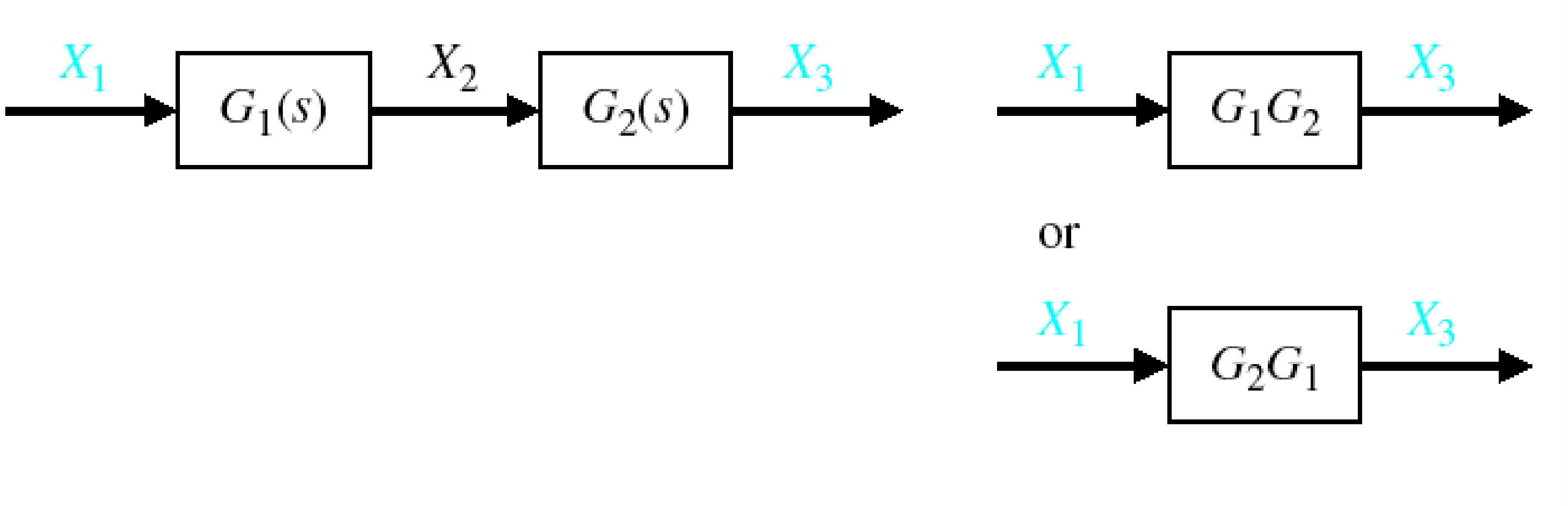
$$\frac{Y_2(s)}{U_1(s)} = G_1 G_2$$

- Blocks in parallel with their outputs added:



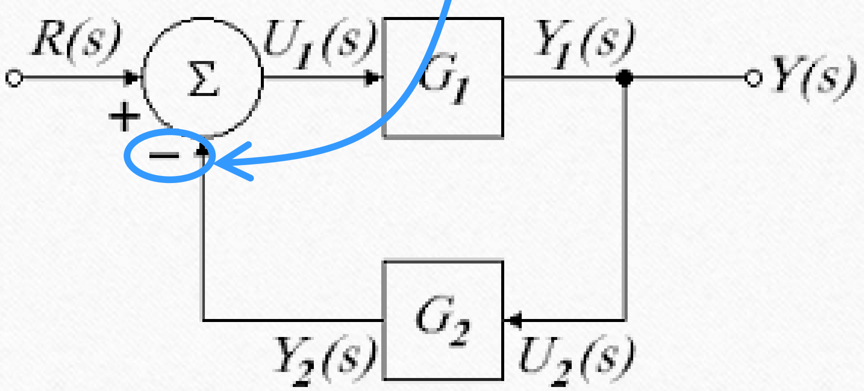
$$\frac{Y(s)}{U(s)} = G_1 + G_2$$

Combining blocks in cascade



Block Diagram Reduction Technique

- ***Single-loop negative feedback***



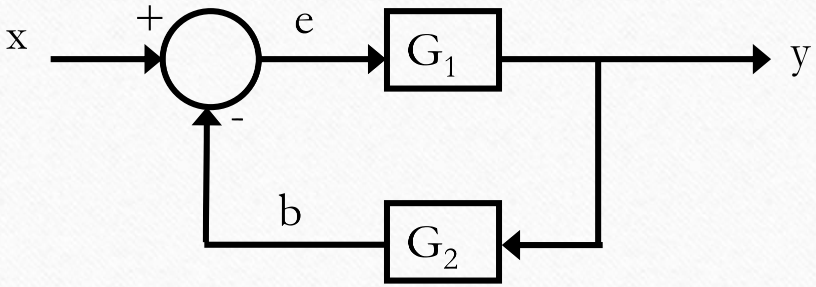
Two blocks are connected in a feedback arrangement so that each feeds into the other.

- **Transfer function**

$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1 G_2}$$

Block Diagram Reduction Technique

- Proof:



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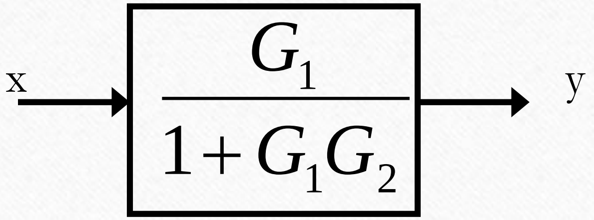


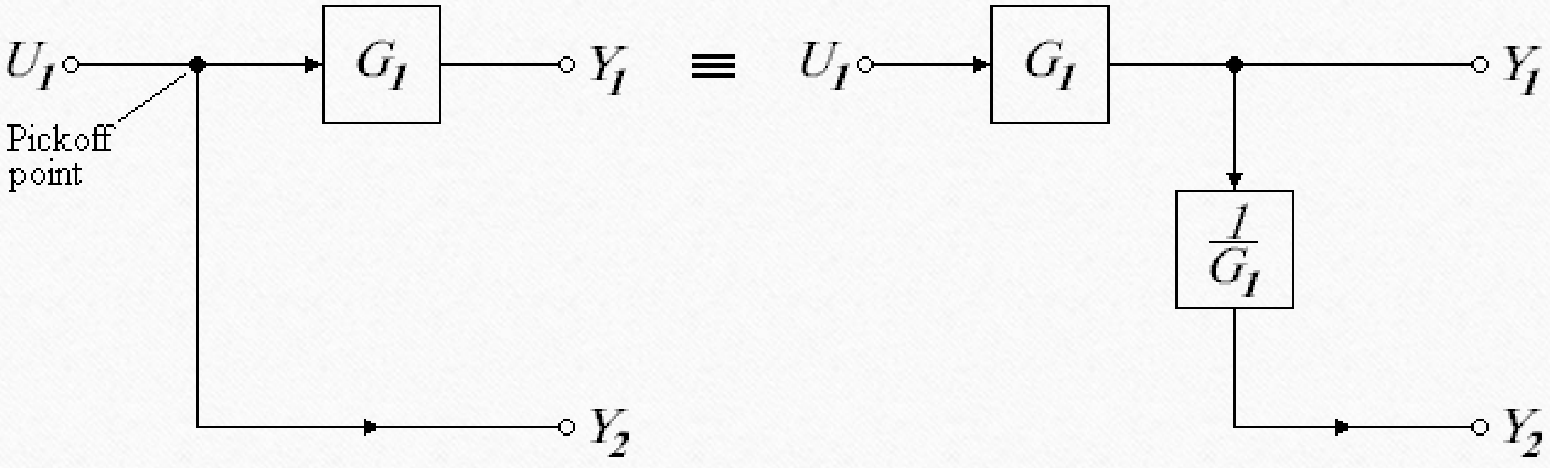
$$y = \frac{G_1}{1+G_1G_2}x$$

$$e = x - b, \quad b = G_2 y, \quad y = G_1 e \quad \Rightarrow \quad y = \frac{G_1}{1+G_1G_2}x$$

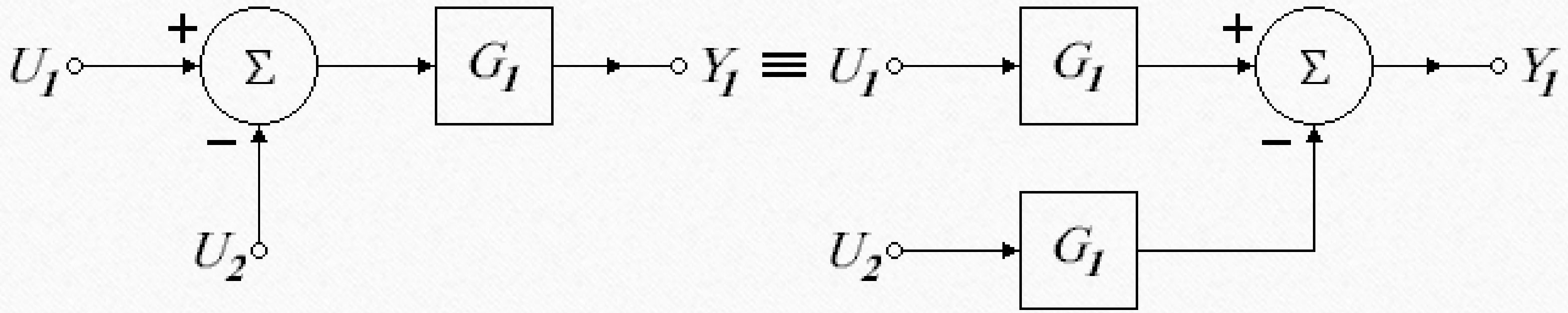
$$e = x - G_2G_1e$$

$$(1+G_1G_2)e = x \;\Rightarrow\; e = \frac{1}{1+G_1G_2}x$$

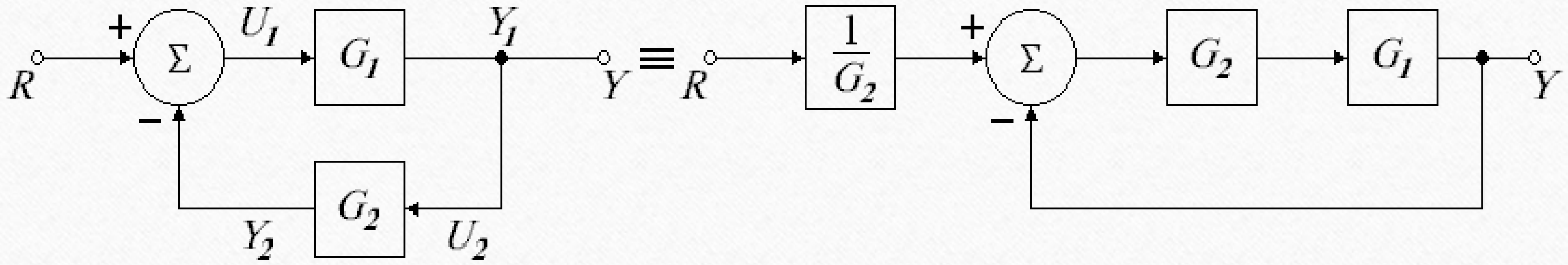
Block Diagram Reduction Technique



Block Diagram Reduction Technique



Block Diagram Reduction Technique



Block Diagram Reduction Technique

Example

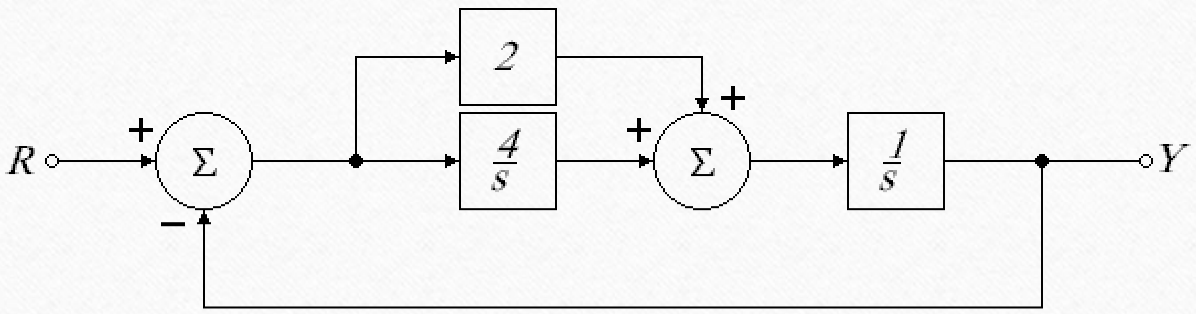


fig. (a)

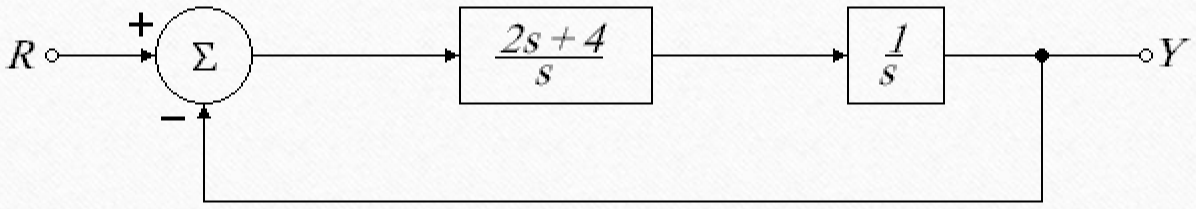


fig. (b)

$$T(s) = \frac{Y(s)}{R(s)}$$

$$T(s) = \frac{\dfrac{2s+4}{s^2}}{1 + \dfrac{2s+4}{s^2}}$$

$$T(s) = \frac{2s+4}{s^2 + 2s + 4}$$

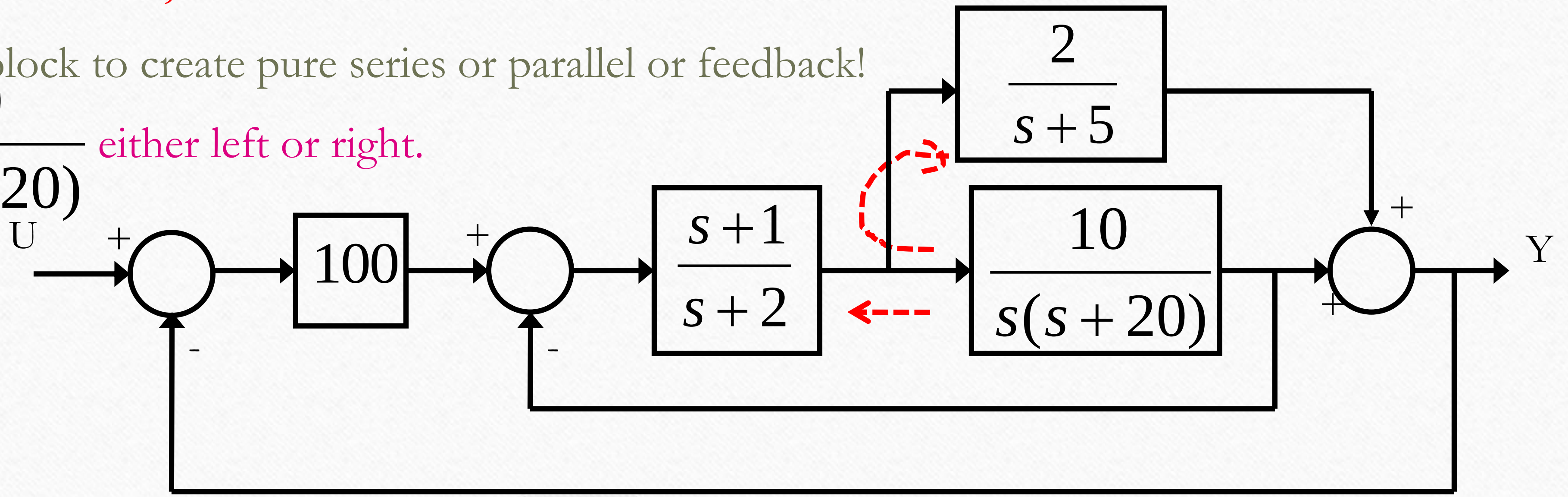
Block Diagram Reduction Technique

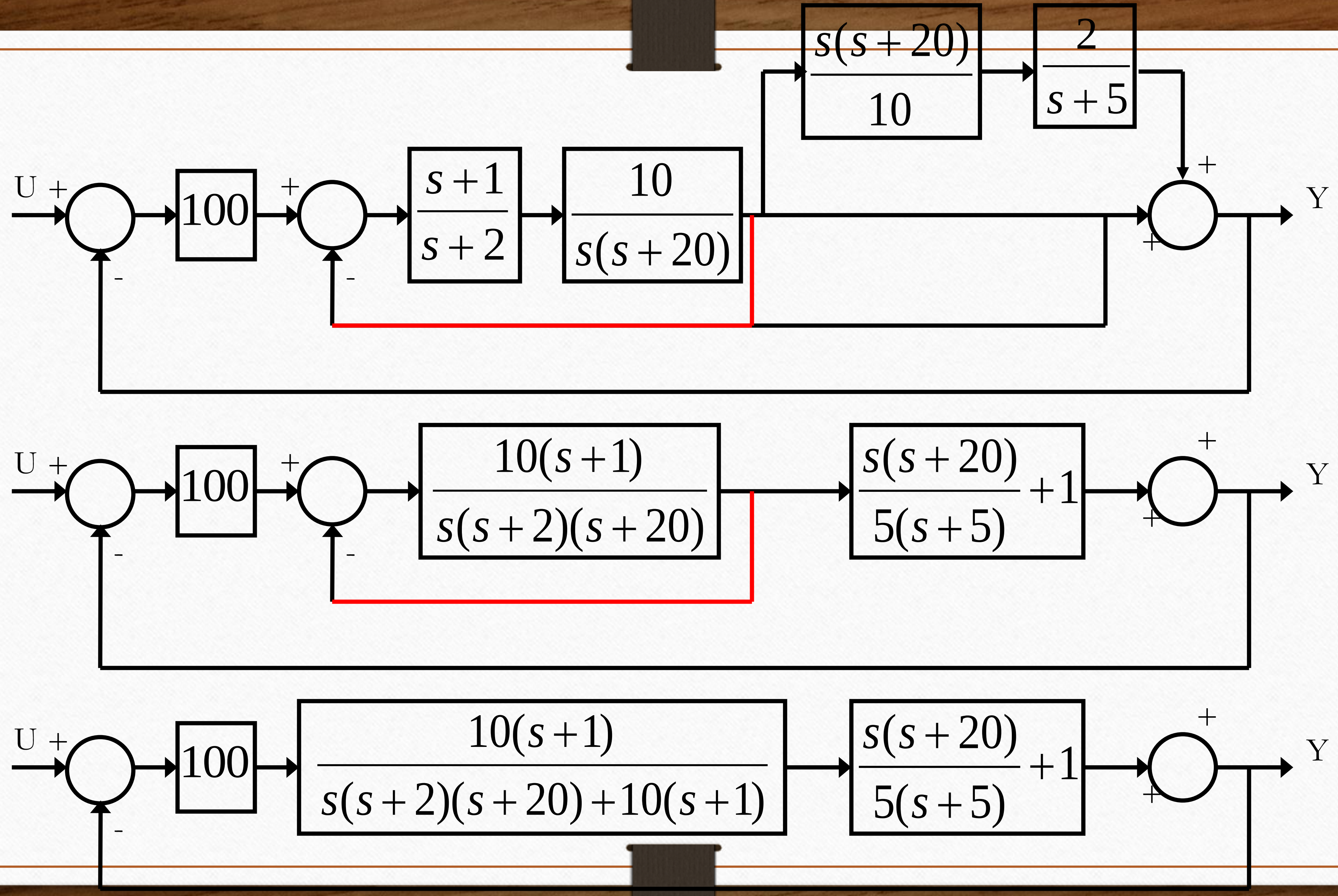
Example 2: Find TF from U to Y:

- No pure series/parallel/feedback
- Needs to move a block, but which one?

Key: move one block to create pure series or parallel or feedback!

So move $\frac{10}{s(s+20)}$ either left or right.

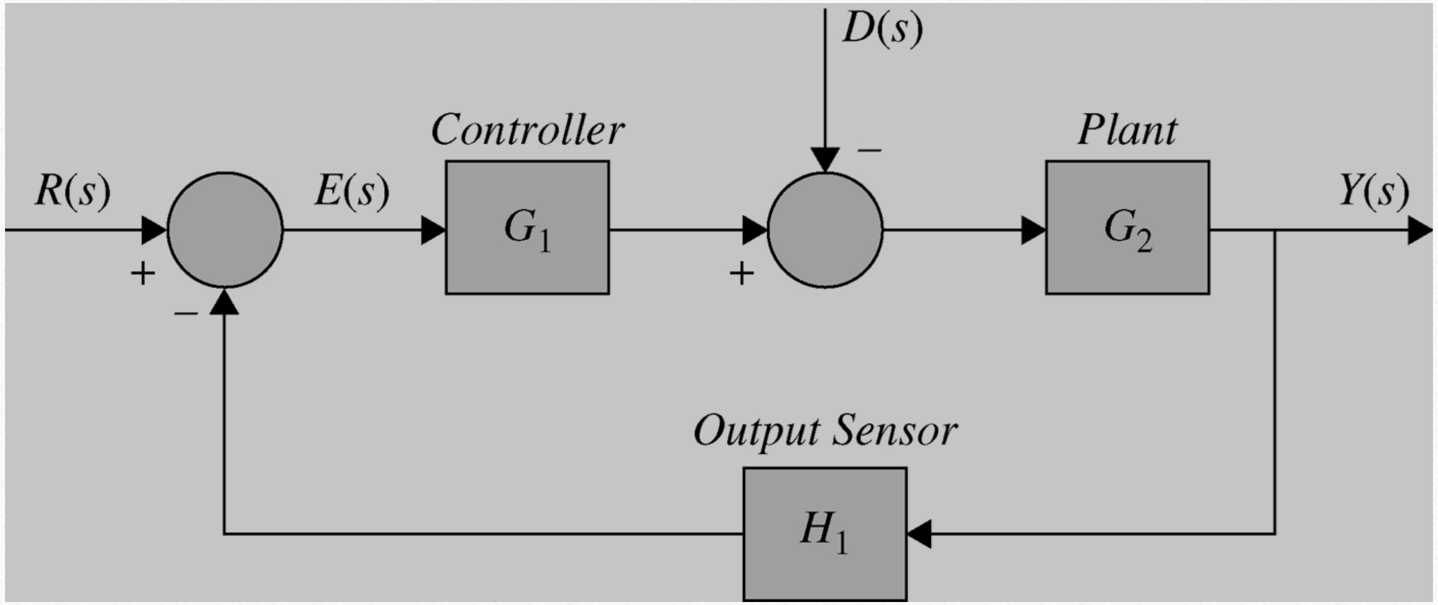




U +
100
+
$\frac{s+1}{s+2}$
$\frac{10}{s(s+20)}$
$\frac{s(s+20)}{10}$
$\frac{2}{s+5}$
+
+
Y
-
-
U +
100
+
$\frac{10(s+1)}{s(s+2)(s+20)}$
$\frac{s(s+20)}{5(s+5)}+1$
+
+
Y
-
-
U +
100
$\frac{10(s+1)}{s(s+2)(s+20)+10(s+1)}$
$\frac{s(s+20)}{5(s+5)}+1$
+
+
Y
-

Block Diagram Reduction Technique

Example



Can use superposition:

First set D=0, find Y due to R

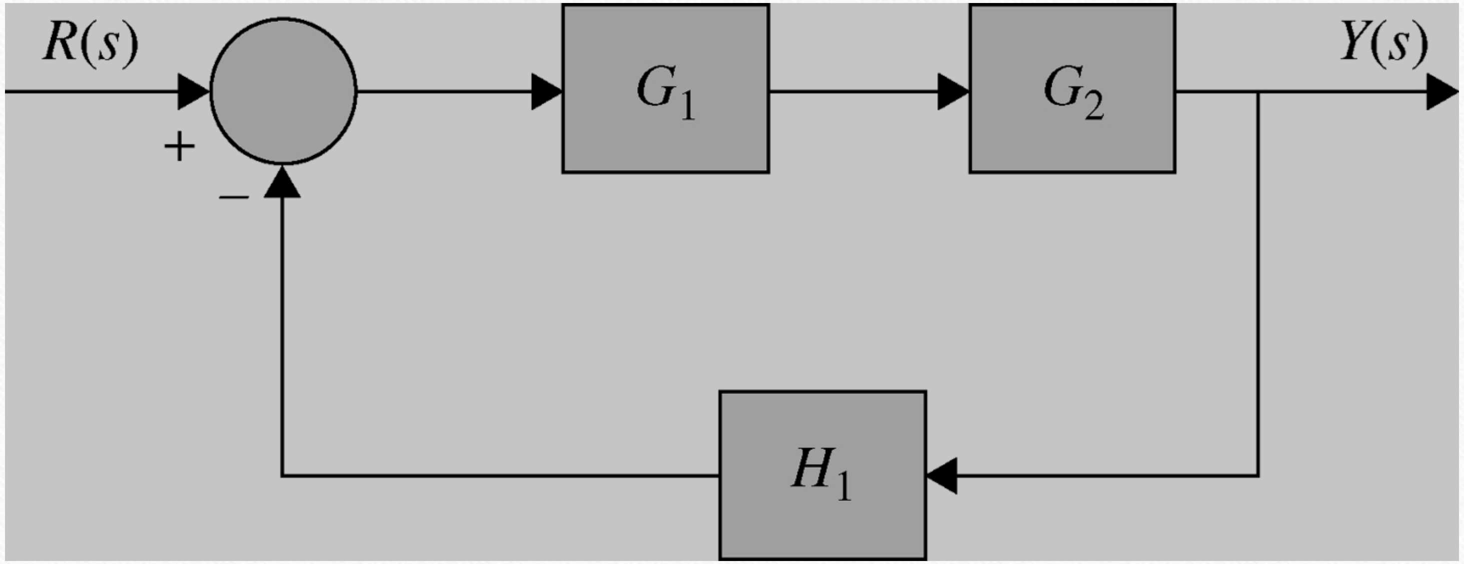
Then set R=0, find Y due to D

Finally, add the two component to get the overall Y

Block Diagram Reduction Technique

First set D=0, find Y due to R

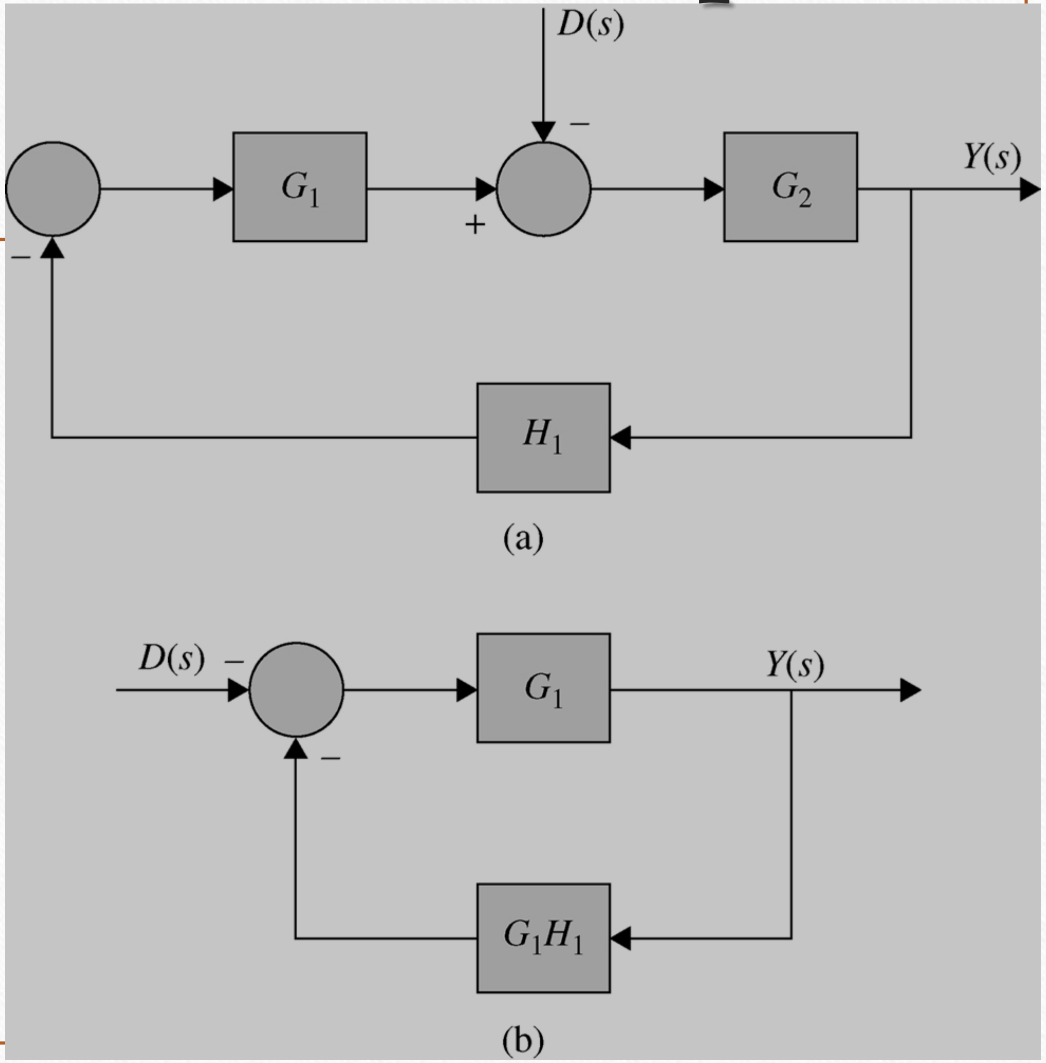
$$Y_1(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_1} R(s)$$



Block Diagram Reduction Technique

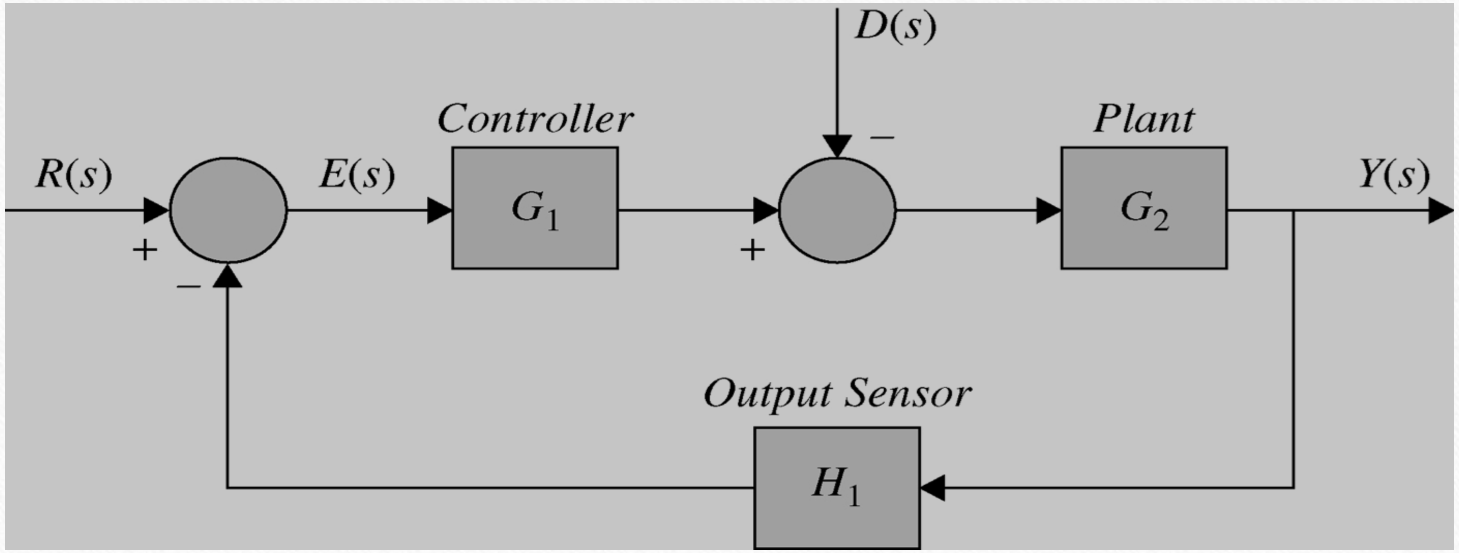
Then set R=0, find Y due to D

$$Y_2(s) = \frac{G_2}{1 + G_1 G_2 H_1}\left(-D(s)\right)$$



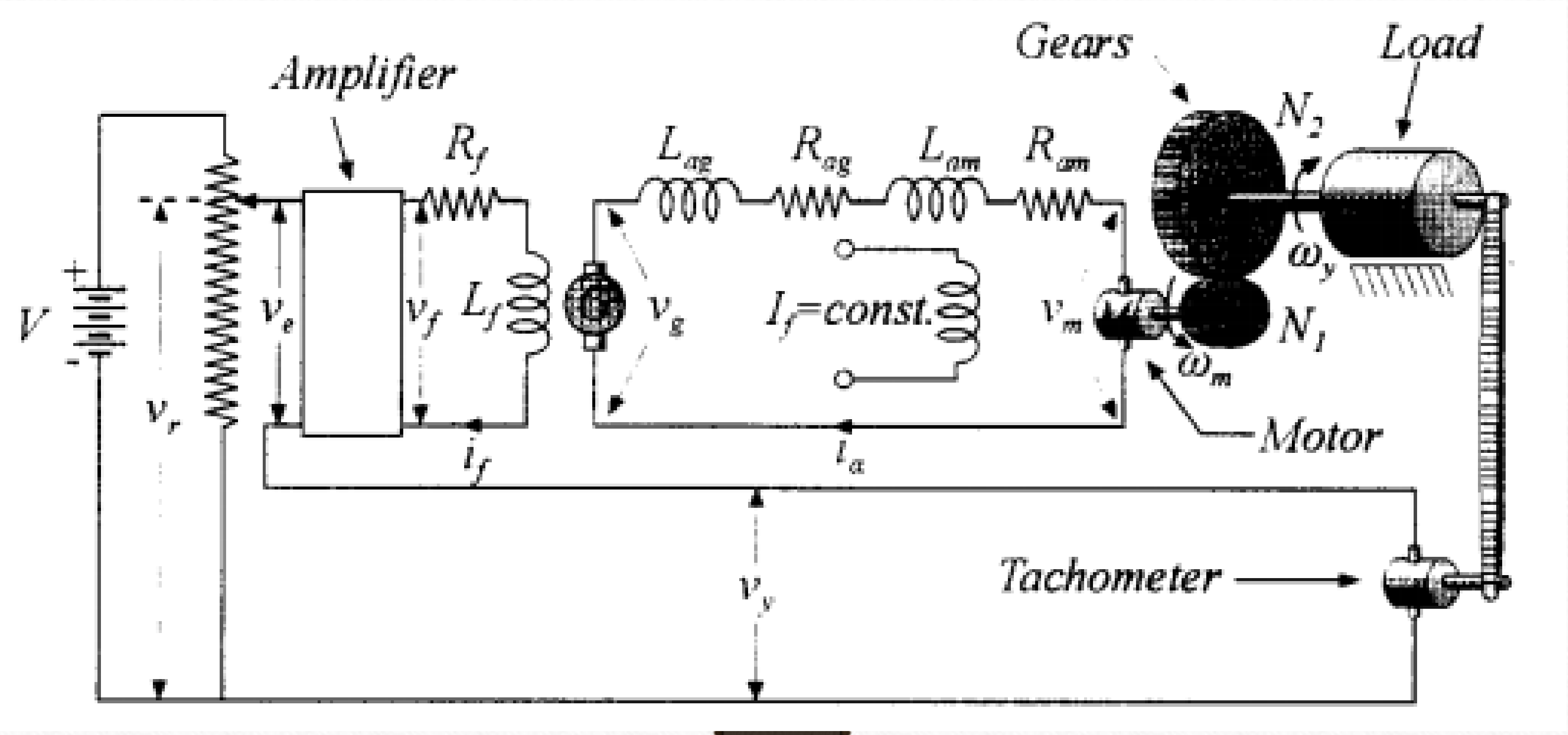
Block Diagram Reduction Technique

Finally, add the two component to get the overall Y



$$Y(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_1} R(s) - \frac{G_2}{1 + G_1 G_2 H_1} D(s)$$

Modeling of Motors



Mathematical Modeling

The equations of the Ward–Leonard layout are as follows . The Kirchhoff's law of voltages of the excitation field of the generator G is

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

The voltage v_g of the generator G is proportional to the current i_f, i.e.,

$$v_g = K_g i_f$$

The voltage v_m of the motor M is proportional to the angular velocity ω_m, i.e.,

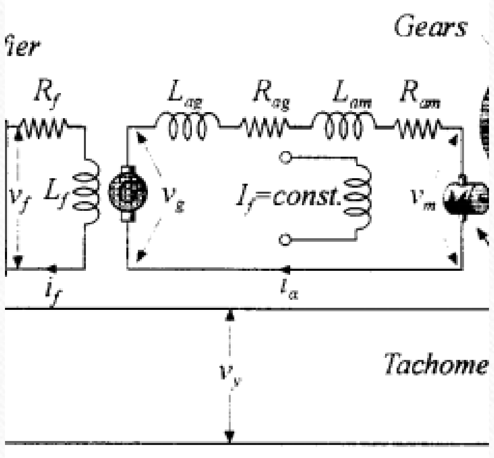
$$v_m = K_b \omega_m$$

The differential equation for the current i_a is

$$R_a i_a + L_a \frac{di_a}{dt} = v_g - v_m = K_g i_f - K_b \omega_m$$

The torque T_m of the motor is proportional to the current i_a

$$T_m = K_m i_a$$



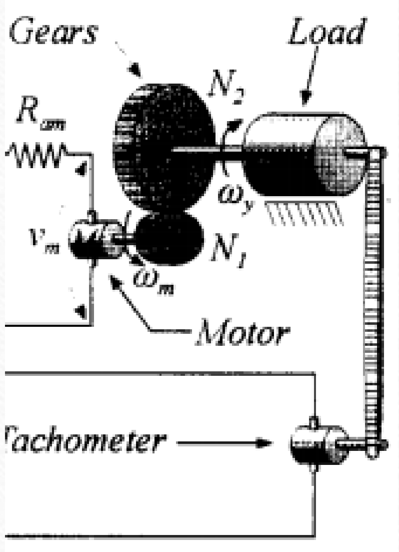
Mathematical Modeling

The equations of the Ward–Leonard layout are as follows . The Kirchhoff's law of voltages of the excitation field of the generator G is

The rotational motion of the rotor is described by

$$J_m^* \frac{d\omega_m}{dt} + B_m^* \omega_m = K_m i_a$$

where J_m*= J_m + $N^2 J_L$ and B_m* =B_m + N^2 B_L, where N = N_1/N_2. Here, J_m is the moment of inertia and B_m the viscosity coefficient of the motor: likewise, for J_L and B_L of the load.



where use was made of the relation

$$\omega_y = N\omega_m.$$

The tachometer equation

$$v_y = K_t \omega_y$$

the amplifier equation

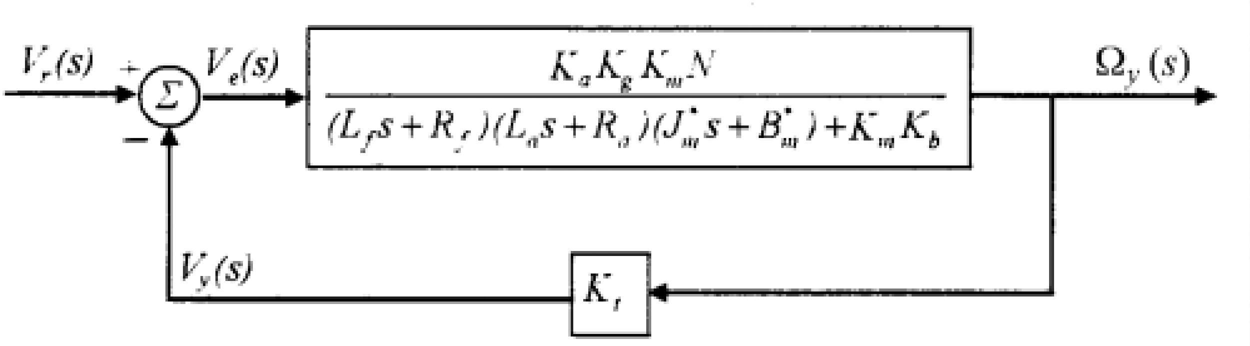
$$v_f = K_a v_e$$

Mathematical Modeling

The mathematical model of the Ward–Leonard layout are as follows .

$$\frac{\Omega_y(s)}{V_f(s)} = \frac{K_g K_m N}{(L_f s + R_f)[(L_a s + R_a)(J_m^* s + B_m^*) + K_m K_b]}$$

$$\frac{\Omega_y(s)}{v_e(s)} = \frac{K_a K_g K_m N}{(L_f s + R_f)[(L_a s + R_a)(J_m^* s + B_m^*) + K_m K_b]}$$



Signal Flow

A **signal-flow graph** is a diagram consisting of nodes that are connected by several directed branches and is a graphical representation of a set of linear relations.

The basic element of a signal-flow graph is a unidirectional path segment called a **branch**

A **loop** is a closed path that originates and terminates on the same node. Two loops are said to be **nontouching** if they do not have a common node

$$T_{ij} = \frac{\sum_k P_{ijk}\,\Delta_{ijk}}{\Delta},$$

P_{ijk} = gain of kth path from variable x_i to variable x_j,

Δ = determinant of the graph,

Δ_{ijk} = cofactor of the path P_{ijk},

$$\Delta = 1 - \sum_{n=1}^{N} L_n + \sum_{\substack{n,\,m \\ \text{nontouching}}} L_n L_m - \sum_{\substack{n,\,m,\,p \\ \text{nontouching}}} L_n L_m L_p + \cdots$$

$\Delta = 1 -$ (sum of all different loop gains)

$+$ (sum of the gain products of all combinations of two nontouching loops)

$-$ (sum of the gain products of all combinations of three nontouching loops)

$+ \cdots$.

The cofactor Δ_{ijk} is the determinant with the loops touching the kth path removed.

Signal Flow

The paths connecting the input *R(s)* and output *Y(s)* are

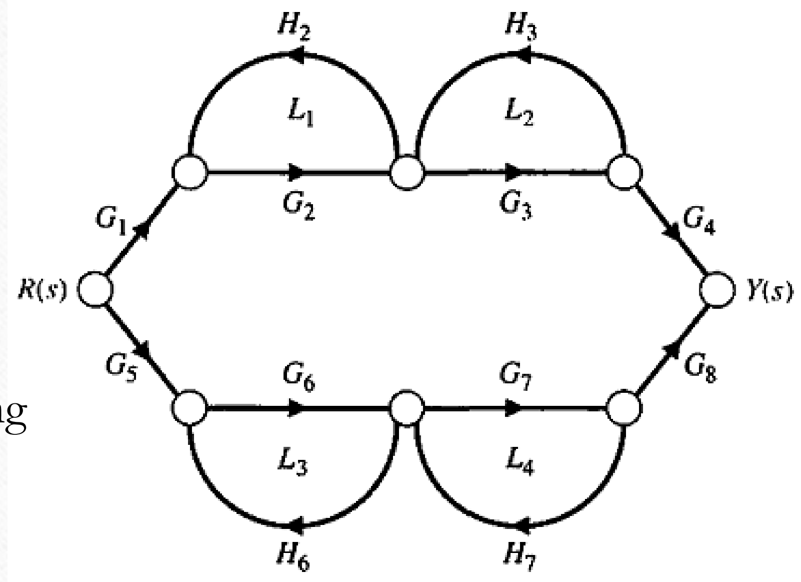
$$P_1 = G_1G_2G_3G_4 \text{ (path 1)} \quad \text{and} \quad P_2 = G_5G_6G_7G_8 \text{ (path 2)}$$

There are four self-loops:

$$L_1 = G_2H_2, \quad L_2 = H_3G_3, \quad L_3 = G_6H_6, \quad \text{and} \quad L_4 = G_7H_7$$

Loops *L1* and L2 do not touch L3 and L4. Therefore, the determinant is

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$$



The cofactor of the determinant along path 1 is evaluated by removing the loops that touch path 1 from Δ. $\Delta_1 = 1 - (L_3 + L_4)$

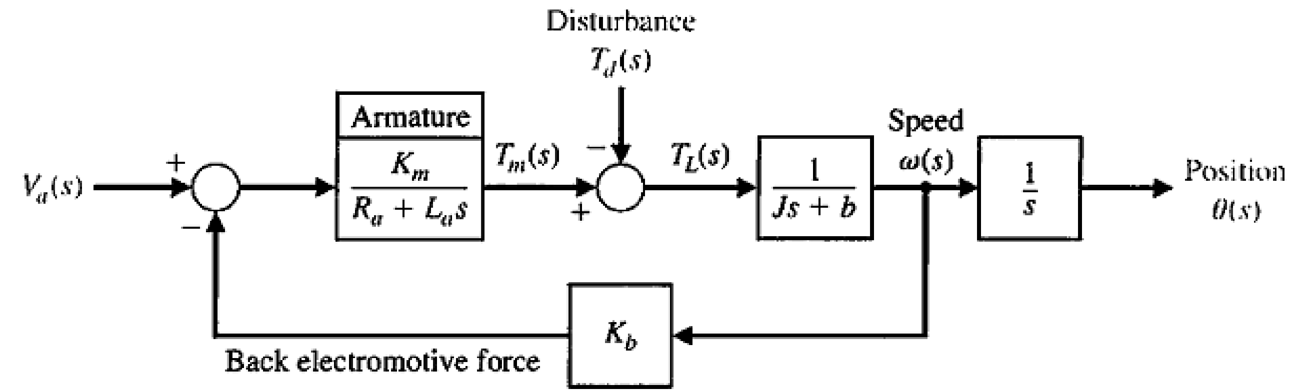
Similarly, the cofactor for path 2 is $\Delta_2 = 1 - (L_1 + L_2)$

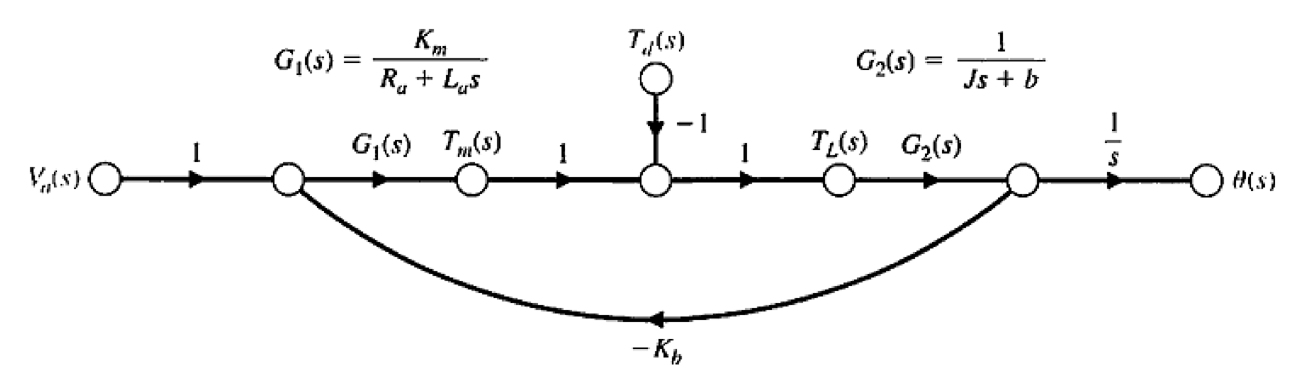
Therefore, the transfer function of the system is

$$\frac{Y(s)}{R(s)} = T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}$$

Signal Flow

The armature-controlled DC motor



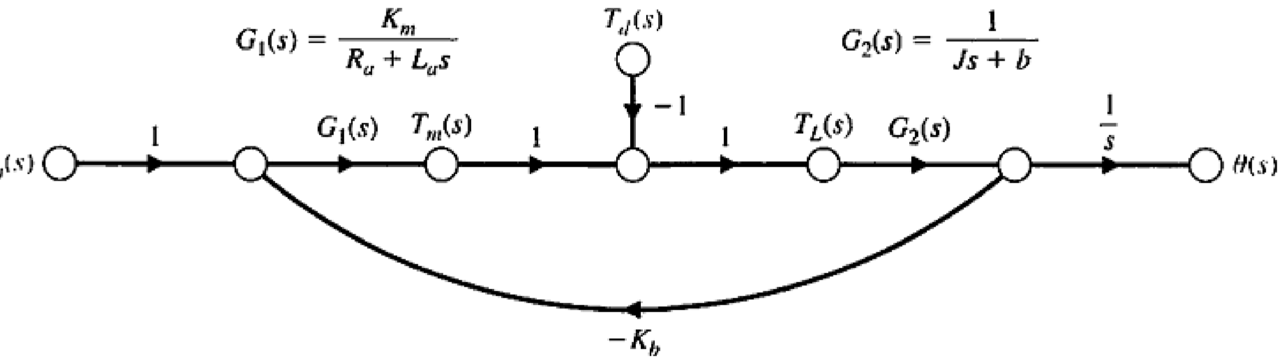


Signal Flow

The armature-controlled DC motor

Using Mason's signal-flow gain formula, transfer function for $\theta(s)/Va(s)$ with $Td(s) = 0$

The forward path is $P1(s)$, which touches the one loop, $L1(s)$, where



$$P_1(s) = \frac{1}{s}G_1(s)G_2(s) \quad \text{and} \quad L_1(s) = -K_b G_1(s)G_2(s).$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$$

$$T(s) = \frac{P_1(s)}{1 - L_1(s)} = \frac{(1/s)G_1(s)G_2(s)}{1 + K_bG_1(s)G_2(s)} = \frac{K_m}{s[(R_a + L_as)(Js + b) + K_bK_m]},$$

Model Examples

- Pulse Width Modulation (PWM)